Center for Assessment Summary of Georgia's Coordinate Algebra End-of-Course Test (EOCT): Areas in Need of Focus

This document summarizes an analysis of mathematics topics from the first iteration of the Coordinate Algebra End-of-Course Test (EOCT), administered Winter 2012. Specifically, this document includes an analysis of students' performance on this EOCT and summarizes the areas where teachers may need to focus their attention to ensure students are learning the content of the new course and the corresponding set of standards. While there are references to specific standards and items as well as curriculum units in this document, it is important to note that these are included to solidify teachers' uses of the resources the Department of Education has developed for the field. This document is to serve as an additional resource to support teachers in their instruction of Coordinate Algebra.

Background

Georgia began its transition to the Common Core State Standards in Mathematics (CCSS-M) in 2010 when the Board of Education adopted these standards. The plan was to have full implementation in kindergarten through grade nine starting in the 2012-2013 academic year. The Georgia Performance Standards (GPS) are a curriculum framework developed by the Department of Education to prepare Georgia teachers for successful implementation of the CCSS-M. As such, Georgia's standards are referred to as the Common Core Georgia Performance Standards (CCGPS).

Students beginning high school mathematics coursework in the 2012-2013 academic year are enrolled in either CCGPS Coordinate Algebra or Accelerated CCGPS Coordinate Algebra/Analytic Geometry A. Coordinate Algebra is a new course for high school mathematics.

Given the new content of the Coordinate Algebra End-of-Course Test, the Georgia Department of Education (DOE) wanted an external third-party analysis of the content of the assessment. The DOE thought such a document could advise the field on the areas of student need, as well as provide another resource to help teachers improve their teaching of the CCGPS in Coordinate Algebra. The DOE contracted the Center for Assessment to conduct this analysis. The Center for Assessment is a 501(c)(3) not-for-profit corporation that seeks to improve student achievement by promoting improved practices in educational assessment and accountability. Since its inception in 1998, the Center has had contracts with approximately two-thirds of the states, and has worked with several additional states, districts, and non-governmental organizations in other capacities. Pamela Paek, Ph.D., a former secondary mathematics teacher and assessment developer, is a senior associate at the Center for Assessment with mathematics content expertise and experience with large-scale assessments. Dr. Paek reviewed and analyzed the new assessment and provided recommendations for how teachers could use the analysis for instructional purposes

Review of Coordinate Algebra Course Content

In the Mathematics High School Course Description document,[1] Coordinate Algebra is described as follows:

> The fundamental purpose of Coordinate Algebra is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Coordinate Algebra uses algebra to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

The underlined portions of the paragraph above emphasize key concepts for this course. They also are some of the areas that were the most troublesome for students on this administration of the EOCT—namely, the idea of deepening, extending, and connecting content across standards and being able to interpret and generalize from different representations (e.g., tables to equations).

Summary of Findings

Rather than provide a list of the standards where students had the most difficulty on the Coordinate Algebra EOCT, a broader analysis of the content across standards was conducted. The reason for this approach was to provide a different framework for how teachers can see how the assessment items and content measured on the EOCT map across the CCGPS for the course. The areas where students were successful included any assessment items that could be solved with "plug and chug"—putting numbers into equations or checking answer choices through trial and error. For the items that required more in-depth conceptual understanding and critical thinking skills, students were not as successful. For students to have strong mastery of the CCGPS, teachers need to focus more on developing deeper understanding of the mathematics for this course.

The main areas where students struggled include:

- Representing functions/equations as graphs or words: understanding what they mean and generalizing interpretations
- Using tables: equations representing relationships, conclusions, and specifying data (generalized), extending findings from the table; generalizing the interpretation of results
- Geometry and the coordinate plane: graphing, applying geometric shapes, and transformation using the coordinate plane
- Exponents, especially functions with time change and graphing and interpreting them

[1] https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_9-12_CoorAlgebra_TeachingGuide.pdf

Each of these is discussed in more detail below.

1. **Representing functions/equations as graphs or words: Understanding what they mean and generalizing interpretations**

   On items where students needed to represent functions/equations as graphs or words, they were challenged when asked for what the answers meant, or to generalize what part of an equation, function, or graph could mean. These types of problems require students to use a greater depth of knowledge, utilizing abstract concepts rather than a concrete algorithmic approach. In short, students need more practice in verbalizing what a function, equation, or graph represents and how to interpret them. This concept crosses almost every standard, so there is not one unit for teachers to focus on. Rather, teachers should help students understand the context of the problems they solve, across the entire Coordinate Algebra course.

   Item 5 from the released Mathematics I EOCT items is an example of interpreting what the results mean. While this example is more explicit in calling for an understanding of what zeroes represent in the context of a word problem, students need to be able to do this for any equation, function, or graph they are given. In other words, they should not just be able to compute the answers: They should also know what they mean beyond a number in the context of the problem they are given.

   The CCGPS says students need to "use graphical representations and knowledge of the context to make judgments" and "interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations." Additionally, they should be able to "create equations to describe situations" and "write, interpret, and translate between various forms of linear equations and inequalities."

   An example from the current CCGPS includes understanding how to use and interpret the line of best fit. This can be done both graphically and as an equation. Students need to know how to interpret the data, graphs, and equations they are given, such as in standards S.ID.3, 5, 6, 7 (Unit 4). The standards that focus on interpretation include:

   - N.Q.1, A.SSE.1.1a, 1b, A.CED.3 (Unit 1)
   - A.REI.5 (Unit 2)
   - F.IF.2, 4-6, 9, F.BF.1a, F.LE.1-3, 5 (Unit 3)
   - S.ID.3, 5, 6, 7 (Unit 4)

   In all of these standards, teachers should focus on developing students' interpretation of content and their ability to see connections across equations and graphs, rather than just algorithmic solutions. Additionally, reviewing the "Key Ideas" for each Main Topic in the

Coordinate Algebra Study Guide[2] (January 11, 2013) will help teach and reinforce some of the knowledge and skills that may not be as obvious if one teaches each standard discretely.

Some of the review and practice items included in the Coordinate Algebra Study Guide can focus students on the types of problems they may see on the EOCT. For instance, see:

- Review Example 1 on pages 104, 110, 141, 151
- Review Example 2 on pages 105, 110, 125, 152
- Review Example 3 on page 153
- EOCT Practice Item 1 on pages 94, 111, 144, 154
- EOCT Practice Item 2 on pages 85, 95, 106, 145, 155
- EOCT Practice Item 3 on pages 52, 86, 99
- EOCT Practice Item 4 on page 131

2. Using tables: Equations representing relationships, conclusions, and specifying data (generalized), extending findings from the table; generalizing interpretation of results

As with graphs, students struggle with effectively using tables to draw conclusions, translating information among tables, equations, and words, and interpreting, describing, and generalizing results. These types of problems require students to use a greater depth of knowledge, utilizing abstract concepts, rather than a concrete algorithmic approach. Students need to become more fluid in their ability to use multiple representations to interpret information from tables. The main areas where this can be practiced are in Units 3 and 4:

- A.REI.11, F.IF.2-6, 9, F.BF.1, F.LE.2, 3 (Unit 3)
- S.ID.2, 5, 6, 6a, 6c, 9 (Unit 4)
- N.Q.1 (Unit 1)

These standards all provide students opportunities to engage in content that requires multiple representations.

In all of these standards, teachers should focus on developing students' interpretation of content and their ability to see the connections across tables and equations, rather than just algorithmic solutions. Additionally, reviewing the "Key Ideas" for each Main Topic in the Coordinate Algebra Study Guide will help teach and reinforce some of the knowledge and skills that may not be as obvious if one teaches each standard discretely.

Some of the practice items included in the Coordinate Algebra Study Guide can focus students on the types of problems they may see on the EOCT. Some items include:

- Review Example 1 on page 124
- Review Example 2 on pages 30, 84, 142, 143

[2] http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Documents/EOCT%20CCGPS%20Coord%20Alg%20Study%20Guide%20Jan%202013.pdf

- EOCT Practice Item 1 on pages 67-68, 77, 99
- EOCT Practice Item 2 on pages 78, 99
- EOCT Practice Item 3 on pages 78, 113

3. **Geometry and the coordinate plane: Graphing, applying geometric shapes, and transformation using the coordinate plane**

Students found translating or rotating figures on the coordinate plane to be relatively straightforward. (This content is the focus of Unit 5 in the CCGPS.) However, they struggled with almost every item that required them to use a coordinate plane with geometry. This includes finding the perimeter or area of a given figure, the distance between points, midpoint, applying geometric transformations with variables in the coordinates (not just numbers), or any other ways of explicitly connecting algebra and geometry through coordinates. This content is the focus of Unit 6 in the CCGPS (G.GPE.4-7).

Some of the practice items included in the Coordinate Algebra Study Guide can focus students on the types of problems they may see on the EOCT. These include:

- Review Example 1 on page 161
- Review Example 2 on page 162
- Review Example 3 on page 163
- Review Example 6 on pages 175-176
- Review Example 7 on page 177
- EOCT Practice Item 1 on page 165
- EOCT Practice Item 2 on page 165
- EOCT Practice Item 3 on pages 166, 179
- EOCT Practice Item 4 on page 180

This approach to geometry is similar to MM1G1 (a, c, and e) of the Mathematics I course, namely determining the distance and midpoint and using the coordinate plane with triangles and quadrilaterals. Mathematics I EOCT released items[3] and the associated commentary[4] can provide some practice and review for what students should be able to do, specifically items 11, 12, 15, and 19. Note that these are examples of ways a student can be assessed on similar standards, but that teachers should look to the CCGPS and Unit 6 particularly, to make sure they are teaching all aspects of this unit, which expands upon the MM1G1 standards from Mathematics I.

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[3] http://archives.gadoe.org/DMGetDocument.aspx/Mathematics%20I%20Released%20Items%20Booklet.pdf?p=6CC6799F8C1371F6A8CA3CF228EE04D756E7F225F369BA4229075E0A06141C30&Type=D

[4] http://archives.gadoe.org/DMGetDocument.aspx/Math%20I%20Released%20Item%20Annotations.pdf?p=6CC6799F8C1371F6D161F49F88A082271381AE497147275FC550E873AD8E44E4&Type=D

Some of the practice items included in the Mathematics I Study Guide[5] can focus students on the types of problems they may see on the EOCT. These include:

- Review Example 1 on pages 116, 122
- Review Example 2 on pages 116-117
- Review Example 3 on page 118
- EOCT Practice Item 1 on pages 119 and 124

Additionally, reviewing the "Key Ideas" for each Main Topic in this Study Guide will help teach and reinforce some of the knowledge and skills that may not be as obvious if one teaches each standard discretely.

4. Exponents, especially functions with time change and graphing and interpreting them

Students struggled with almost every item that included exponents. This was most problematic with items that included word problems where students needed to translate a word problem into an equation, and sometimes solve that equation. It also translated into interpreting sequences, which is a form of generalizing. Students tended to struggle with generalizing in all areas.

While Unit 3 focuses on exponential functions, the following standards also include students' learning of exponents:

- SSE1, 1a, 1b, MCC9-12, A.CED.1, 2 (Unit 1)
- A.REI.3 (Unit 2)
- A.REI.10, 11, I.IF.1, 2, 4-7, 9 (Unit 3)
- MCC9-12F.BF.1, 1a, 1b, 2, 3 (Unit 3)
- F.LE.1, 1a, 1b, 1c, 2, 3, 5 (Unit 3)

Teachers should find ways to connect and build on the foundations of exponents across these standards and units to help solidify student understanding of them. Since students struggle with uses of multiple representations and interpretations, teachers must include in their teaching how students can become facile with exponents—what they mean and how they can be used to solve problems. Additionally, reviewing the "Key Ideas" for each Main Topic in the Coordinate Algebra Study Guide will help teach and reinforce some of the knowledge and skills that may not be as obvious if one teaches each standard discretely.

Some of the practice items included in Coordinate Algebra Study Guide can focus students on the types of problems they may see on the EOCT. These include:

- Review Example 1 on pages 24, 30, 83, and 104

5 http://archives.gadoe.org/DMGetDocument.aspx/GaEOCT%20Math%20I%20Study%20Guide%20%20.pdf?p=6CC6799F8C1371F6EEB5636BAD102294ECAA47390F67FE9BD1EF2D6FEAD07DFC&Type=D

- EOCT Practice Item 2 on pages 25 and 85

Summary

The Coordinate Algebra EOCT consists of items that reflect the expectations stated in the CCGPS. These items assess deep understanding of the content, by asking students to interpret the meaning of their solutions and to use multiple types of representations to describe relationships. Interpreting and describing solutions in multiple ways are fundamental to mathematics mastery, not just for this course, but also for strong mathematical understanding in general. Math should **not** be taught in discrete pieces or as process skills. Rather, mathematics should be taught by teaching concepts in a more unified manner, so students learn how mathematical concepts work together and can be applied, through the use of the eight mathematical practices as described in the CCGPS. It is critical for teachers to teach this course in ways that encourage students' deeper understanding through the practice of reflection and planning in solving complex problems, interpreting and describing solutions, and reinforcing how the content of the course builds on previous knowledge.